ECE 2300 Digital Logic and Computer Organization Fall 2024

Topic 5: Number Systems

School of Electrical and Computer Engineering
Cornell University

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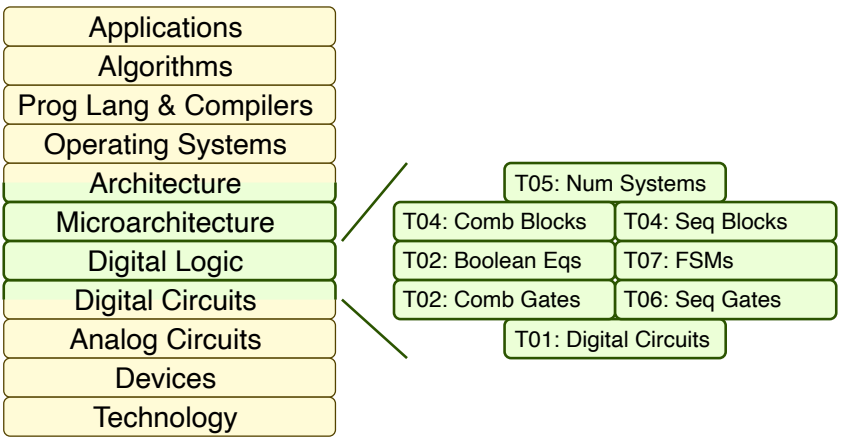


1. Unsigned Binary Number System

$b_3b_2b_1b_0$	v
0000	
0001	
0010	
0011	
0100	
0101	
0110	
0111	
1000	
1001	
1010	
1011	
1100	
1101	
1110	
1111	

$$v = b_3 \times 2^3 + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

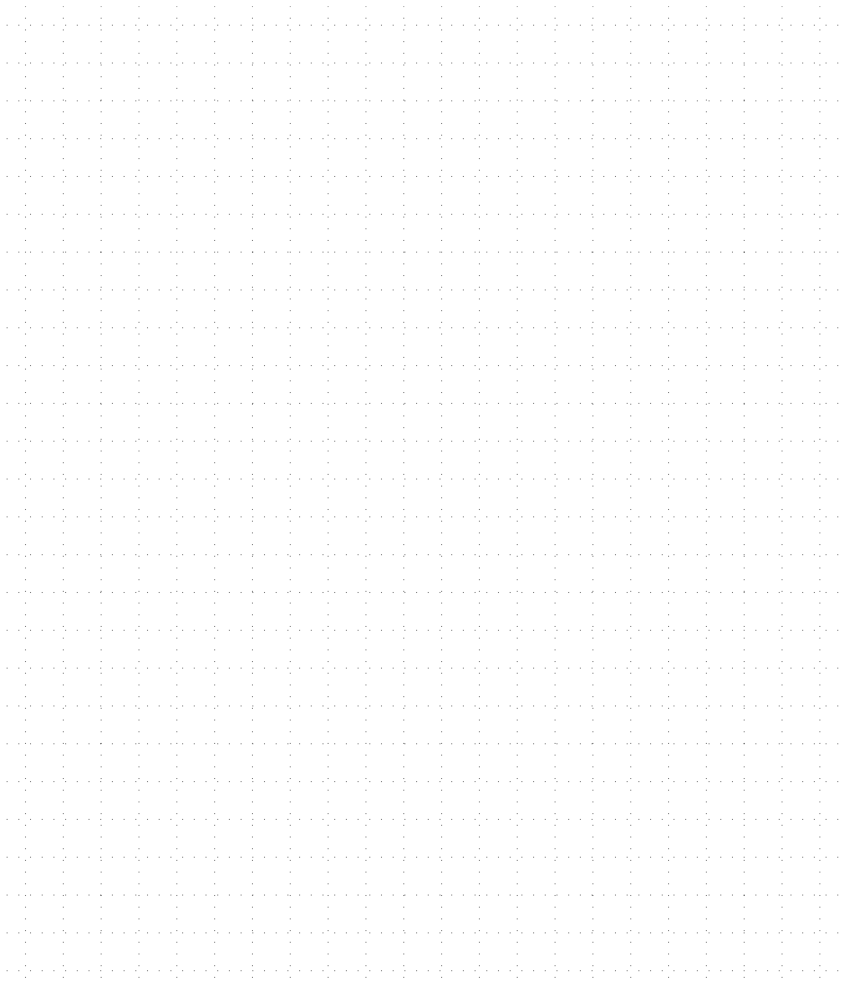
How do we represent $v = 13$?

What does $b = 1010$ represent?

How do we represent $v = 7$?

What does $b = 1110$ represent?

Implement an addsub unit for unsigned binary numbers



2. Sign Magnitude Number System

$b_3b_2b_1b_0$	v
0000	
0001	
0010	
0011	
0100	
0101	
0110	
0111	
1000	
1001	
1010	
1011	
1100	
1101	
1110	
1111	

$$v = -1^{b_3} \times (b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0)$$

How do we represent $v = -5$?

What does $b = 1100$ represent?

How do we represent $v = -7$?

What does $b = 0101$ represent?

Complementing a signed magnitude number

- Simply invert the sign bit (i.e., the most significant bit)

Implement an addsub unit for signed magnitude numbers

3. Two's Complement Number System

$b_3b_2b_1b_0$	v
0000	
0001	
0010	
0011	
0100	
0101	
0110	
0111	
1000	
1001	
1010	
1011	
1100	
1101	
1110	
1111	

$$v = b_3 \times -(2^3) + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

How do we represent $v = -4$?

What does $b = 1110$ represent?

How do we represent $v = -1$?

What does $b = 0101$ represent?

Complementing a two's complement number

v_{bin}	$b_3b_2b_1b_0$	v_{2c}	$-v_{2c}$			
0	0000	0	0	0000	0	16-0 = 16
1	0001	1	-1	1111	15	16-1 = 15
2	0010	2	-2	1110	14	16-2 = 14
3	0011	3	-3	1101	13	16-3 = 13
4	0100	4	-4	1100	12	16-4 = 12
5	0101	5	-5	1011	11	16-5 = 11
6	0110	6	-6	1010	10	16-6 = 10
7	0111	7	-7	1001	9	16-7 = 9
8	1000	-8	-8	1000	8	16-8 = 8
9	1001	-7	7	0111	7	16-9 = 7
10	1010	-6	6	0110	6	16-10 = 6
11	1011	-5	5	0101	5	16-11 = 5
12	1100	-4	4	0100	4	16-12 = 4
13	1101	-3	3	0011	3	16-13 = 3
14	1110	-2	2	0010	2	16-14 = 2
15	1111	-1	1	0001	1	16-15 = 1

- Two's complement of a 4-bit number v is $2^4 - v$
- Implementing this in hardware requires 5-bit subtractor (maybe?)

Implement an addsub unit for two's complement numbers

4. Fixed Point Number System

$b_3b_2b_1b_0$	v
0000	
0001	
0010	
0011	
0100	
0101	
0110	
0111	
1000	
1001	
1010	
1011	
1100	
1101	
1110	
1111	

$$v = b_3 \times 2^1 + b_2 \times 2^0 + b_1 \times 2^{-1} + b_0 \times 2^{-2}$$

How do we represent $v = 1.5$?

What does $b = 1011$ represent?

How do we represent $v = 3.25$?

How do we represent $v = 3.33$?

5. Floating Point Number System

$b_3b_2b_1b_0$	v
0000	0.2500
0001	0.3125
0010	0.3750
0011	0.4375
0100	0.5000
0101	0.6250
0110	0.7500
0111	0.8750
1000	1.0000
1001	1.2500
1010	1.5000
1011	1.7500
1100	2.0000
1101	2.5000
1110	3.0000
1111	3.5000

$$v = 2^{(b_3 \times 2^1 + b_2 \times 2^0) - 2} \times (1 + b_1 \times 2^{-1} + b_0 \times 2^{-2})$$

- b_3b_2 is called the exponent part
  - b_3b_2 = 00 represents 0.25 (subrange 0.25–0.50)
  - b_3b_2 = 01 represents 0.50 (subrange 0.50–1.00)
  - b_3b_2 = 10 represents 1.00 (subrange 1.00–2.00)
  - b_3b_2 = 11 represents 2.00 (subrange 2.00–4.00)
- b_1b_0 is called the fraction part
  - fraction part divides up subrange

How do we represent $v = 0.625$?

How do we represent $v = 0.33$?