IMPLICANT

PRODUCT of multiple TERMS

$\bar{A}B$, $A\bar{B}\bar{C}$, B

MINTERM

PRODUCT INVOLVING ALL OF THE INPUTS TO THE FUNCTION

$A\bar{B}\bar{C}$ IS MINTERM for FUNCTION of THREE VARIABLES

$\bar{A}B$ IS NOT A MINTERM for FUNCTION OF 3 VARIABLES

PRIME IMPLICANT

CANNOT BE COMBINED WITH ANY OTHER IMPLICANTS IN EQUATION TO FORM A NEW IMPLICANT w/ FEWER VARIABLES

$$F = \underbrace{\bar{A}BC + \bar{A}B\bar{C}}_{\text{NOT prime}} + \underbrace{A\bar{B}\bar{C}}_{\text{prime}}$$

$F = \bar{A}B + A\bar{B}\bar{C}$ → BOTH prime

C \ AB	00	01	11	10
0		$\boxed{1}$		$\boxed{1}$
1		$\boxed{1}$		

→ $A\bar{B}\bar{C}$

→ $\bar{A}B$

ESSENTIAL PRIME IMPLICANT

Prime IMPLICANT THAT COVERS AN OUTPUT of FUNCTION THAT NO OTHER prime implicant IS ABLE TO COVER

Cost

ASSUME COST IS APPROXIMATED BY FEWEST IMPLICANTS, "MINIMAL cover"

SLIDE 18

AFTER ROW DOMINANCE WE ARE LEFT WITH

	$A'B'$	$C'D$	$A'D$	$A'C$
2	X			X
5		X	X	
7			X	X

COLUMN $A'D$ DOMINATES $C'D$

	$A'B'$	$A'D$	$A'C$
2	X		X
5		X	
~~7~~		~~X~~	~~X~~

ROW 7 DOMINATES ROW 5

	$A'B'$	$A'D$	$A'C$
2	X		X
~~5~~		~~X~~	

$A'D$ IS AN ESSENTIAL PRIME IMPLICANT

	$A'B'$	$A'C$
2	X	X

$A'B'$ AND $A'C$ HAVE COLUMN CO-DOMINANCE SO WE CAN CHOOSE EITHER ONE TO CROSS OUT

SO WE ARE LEFT WITH TWO OPTIMAL SOLUTIONS

$$F = A'D + A'B'$$

$$F = A'D + A'C$$

NOTE THAT $F = A'C + C'D$ IS ALSO A MINIMAL COVER BUT IT WAS ELIMINATED THROUGH COLUMN DOMINANCE.

TAKEAWAY: THERE CAN BE MANY OPTIMAL SOLUTIONS. QM IS GUARANTEED TO FIND <u>A</u> OPTIMAL SOLUTION BUT NOT <u>ALL</u> OPTIMAL SOLUTIONS

QM Branch-and-Bound Algorithm

1. Generate prime implicants

2. Construct prime implicant table

3. Reduce prime implicant table

   3A. Remove essential prime implicants
   3B. Row dominance
   3C. Column dominance
   3D. Goto 3A until no further reductions possible

4. Solve prime implicant table

   4A. Heuristically choose a prime implicant

   4B. Assume chosen PI is in minimal cover
   (cross out column + corresponding intersected rows)

   4C. Goto 3

   4D. Assume chosen PI is not in minimal cover
   (cross out column but <u>not</u> intersected rows)

   4E. Goto 3

   4F. Return minimum solution returned from 4C and 4E

If at any time size of current solution (ie number of product terms) is greater than best solution found so far, return from current recursive step immediately

	$A'C$	$B'C$	$A'B$	BC'	AB'	AC'
3	x	x				
5			x	x		
7	x		x			
9					x	x
11		x			x	
13				x		x

CHOOSE $A'C$, ASSUME IN MIN COVER

CURR SOLUTION $= \{A'D', A'C\}$

	$B'C$	$A'B$	BC'	AB'	AC'
5		x	x		
9				x	x
11	x			x	
13			x		x

NO ESSENTIAL PRIME IMPLICANTS

NO ROW DOMINANCE

COL DOMINANCE: $BC' > A'B$, $AB' > B'C$

	BC'	AB'	AC'
5	x		
9		x	x
11		x	
13	x		x

ESSENTIAL PRIME IMPLICANTS: BC', AB'

FINAL SOLUTION $= \{A'D', A'C, BC', AB'\}$

	$A'C$	$B'C$	$A'B$	BC'	AB'	AC'
3	x	x				
5			x	x		
7	x		x			
9					x	x
11		x			x	
13				x		x

CHOOSE $A'C$, ASSUME NOT IN MIN COVER

CURR SOLUTION $= \{A'D'\}$

	$B'C$	$A'B$	BC'	AB'	AC'
3	x				
5		x	x		
7		x			
9				x	x
11	x			x	
13			x		x

ESSENTIAL PRIME IMPLICANTS: $B'C$, $A'B$

CURR SOLUTION $= \{A'D', B'C, A'B\}$

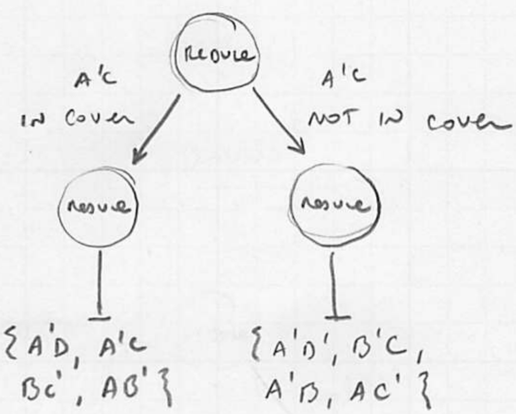
	$B'C$	AB'	AC'
9		x	x
13	x		x

NO ROW DOMINANCE

COL DOMINANCE: $AC' > BC'$, $AC' > AB'$

FINAL SOLUTION $= \{A'D', B'C, A'B, AC'\}$

BRANCH AND BOUND SEARCH TREE



ANOTHER EXAMPLE

	A	B	C	D	E	F
0	x		x			
1	x	x		x		
2				x	x	
3		x	x		x	x
4				x		x

ASSUME WE HAVE THIS IMPLICANT TABLE AFTER STEPS 1, 2, 3

THIS IS A CYCLIC COVER!

CHOOSE A IS IN MIN COVER

	~~A~~	B	C	D	E	F
~~0~~	~~x~~		~~x~~			
~~1~~	~~x~~	~~x~~		~~x~~		
2				x	x	
3		x	x		x	x
4				x		x

=>

	~~B~~	~~C~~	D	E	F
2			x	x	
3	x	x		x	x
4			x		x

=>

	D	E	F
2	x	x	
3		x	x
4	x		x

COL DOM: E > B, E > C

ANOTHER CYCLIC COVER!

CHOOSE D IN SOLUTION

	~~D~~	E	F
~~2~~	~~x~~	~~x~~	
3		x	x
~~4~~	~~x~~		~~x~~

SOL = {A, D, E} OR {A, D, F}

CHOOSE D ϕ IN SOLUTION

	~~D~~	E	F
2	x	x	
3		x	x
4	x		x

=>

	E	F
2	x	
~~3~~	~~x~~	~~x~~
4		x

ROW DOM 3 > 2, 3 > 4

SOL = {A, E, F}

CHOOSE A IS NOT IN THE MIN COVER

	A	B	C	D	E	F
0	x		x			
1	x	x		x		
2				x	x	
3		x	x		x	x
4				x		x

$\Longrightarrow$

	B	C	D	E	F
0		x			
1	x		x		
2			x	x	
3	x	x		x	x
4			x		x

$\Longrightarrow$

	B	D	E	F
1	x	x		
2		x	x	
4		x		x

C IS ESSENTIAL PRIME

COL DOMINANCE
D > B, D > E, D > F

SOL = {C, D}

SO BEST SOLUTION IS {C, D} w/ 2 PRIME IMPLICANTS

WHAT IF WE STARTED BY CHOOSING A IS NOT IN THE MIN COVER AND THEN TRIED CHOOSING A IN THE MIN COVER?

ABOVE WOULD BE THE SAME, THEN WHEN WE CHOOSE A IN THE MIN COVER WE WOULD START SIMILAR TO BEFORE:

	A	B	C	D	E	F
~~0~~	x		x			
~~1~~	x	x		x		
2				x	x	
3		x	x		x	x
4				x		x

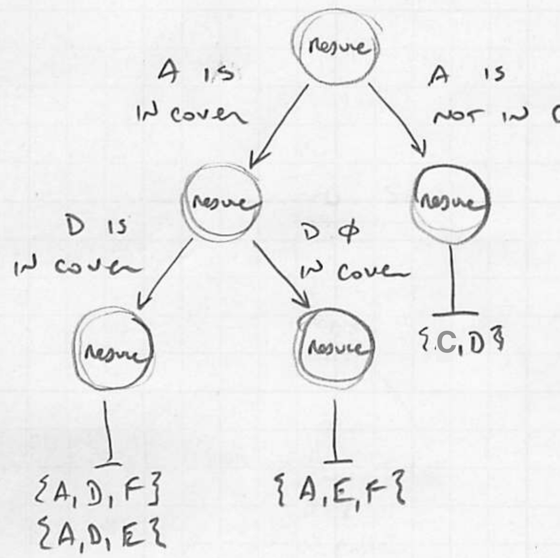
$\Longrightarrow$

	B	C	D	E	F
2			x	x	
3	x	x		x	x
4			x		x

WE CAN ACTUALLY STOP RIGHT AWAY. WE ALREADY HAVE A IN THE SOLUTION THE BEST CASE WOULD BE TO COVER THE REMAINING ROWS WITH A SINGLE PRIME IMPLICANT, BUT THAT MEANS SOLUTION HAS 2 PI, AND WE ALREADY HAVE A SOLUTION WITH 2 PI!

BY USING A BOUND WE CAN STOP SEARCHING EARLY WHILE STILL GUARANTEEING OPTIMALITY.

BRANCH AND BOUND SEARCH TREE



- BRANCH ON CYCLIC COVERS
- BOUND TO REDUCE SEARCH TIME