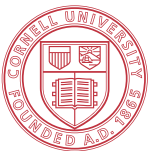




ECE 6775
High-Level Digital Design Automation
Fall 2025

Control Flow Graph



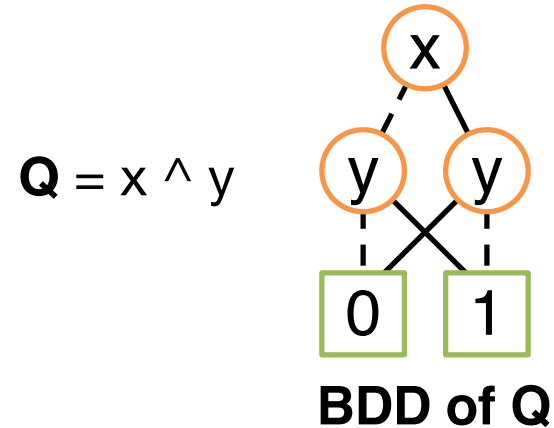
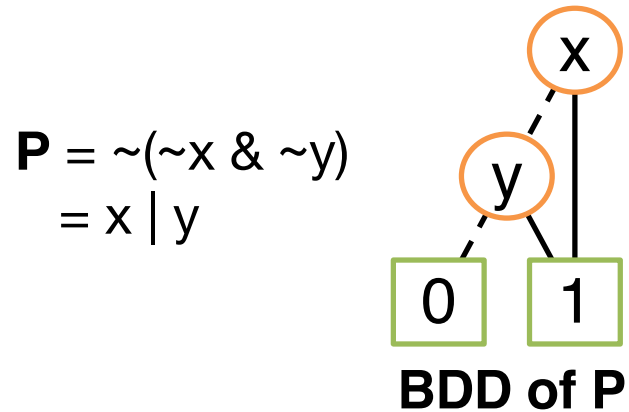
Cornell University



Announcements

- ▶ HW 1 due tomorrow
- ▶ Lab 2 will be released soon

Recap: Equivalence Checking using BDDs

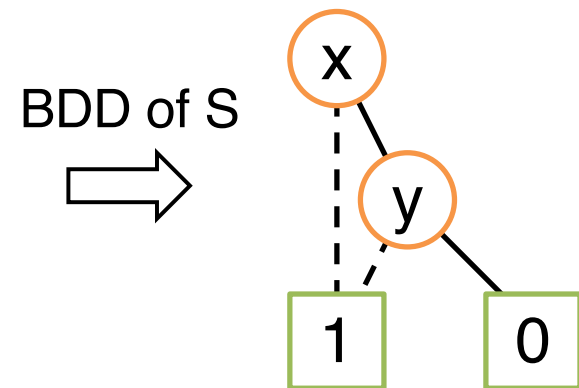


Checking equivalence of P and Q by constructing the BDD of $(P == Q)$

Let S denote the function of $(P == Q)$, i.e., $P \text{ XNOR } Q$

$$S_{x=1} = P_{x=1} \text{ XNOR } Q_{x=1} = 1 \text{ XNOR } y' = y'$$

$$S_{x=0} = P_{x=0} \text{ XNOR } Q_{x=0} = y \text{ XNOR } y = 1$$



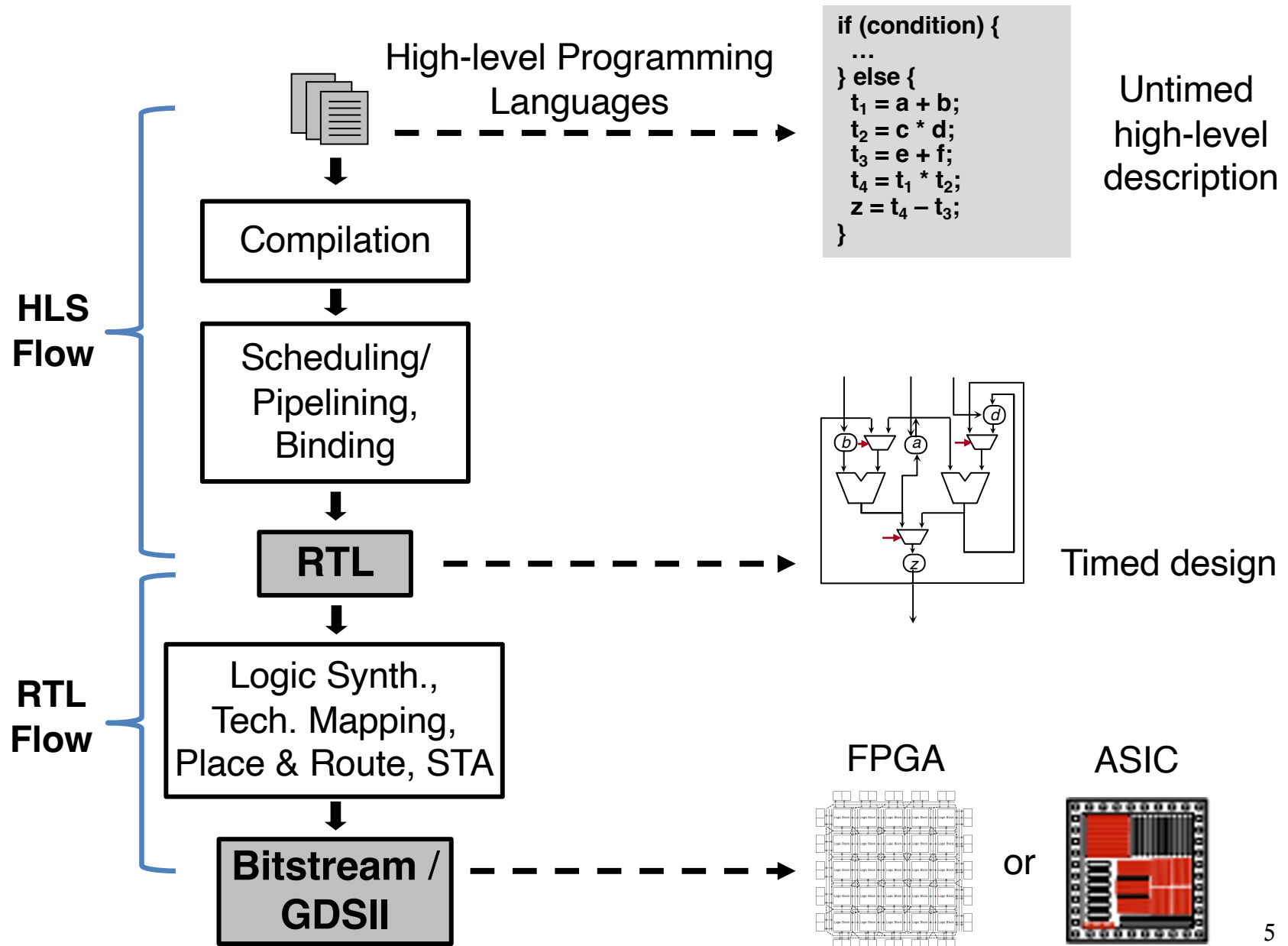
Boolean Satisfiability Problem (SAT)

- ▶ Modern equivalence checking relies heavily on SAT solvers, complemented by other techniques
- ▶ Given a Boolean function $F(x_1, x_2, \dots, x_n)$, find an assignment to x_i 's to make F evaluate to 1
 - If such assignment exists, F is satisfiable; Otherwise, F is unsatisfiable
 - F is typically expressed in Conjunctive Normal Form (CNF)
- ▶ Example: $(x + y + z) (x' + y' + z) (x' + y' + z')$
 - A satisfying assignment: $x=1, y=0, z=1$
- ▶ First NP-complete problem (Cook-Levin theorem)
- ▶ SAT solvers have made significant progress in scalability
 - From toy problems with 100-200 variables (early 90s)
 - To industrial applications with 1M+ variables, 10M+ clauses (now)

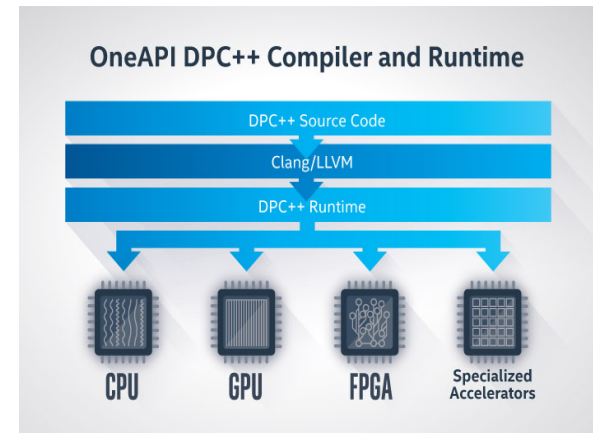
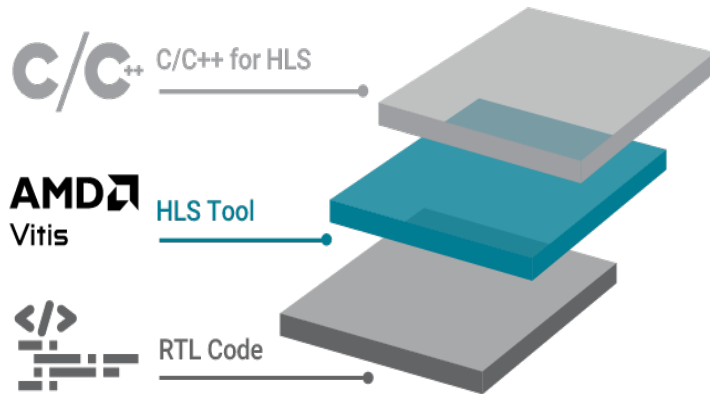
Agenda

- ▶ A typical HLS compilation flow
 - Front-end compilation and intermediate representation
- ▶ Basics of control data flow graph
 - Basic blocks
 - Control flow graph
- ▶ Dominance relation
 - Finding loops

Accelerator Design Flow with HLS



HLS for FPGAs: Entering the Mainstream



Press Releases Date: 08/07/2018

- *Achronix and Mentor Partner to Provide Link Between High-Level Synthesis and FPGA*
- *Achronix Speedcore eFPGAs fully supported by Catapult HLS*

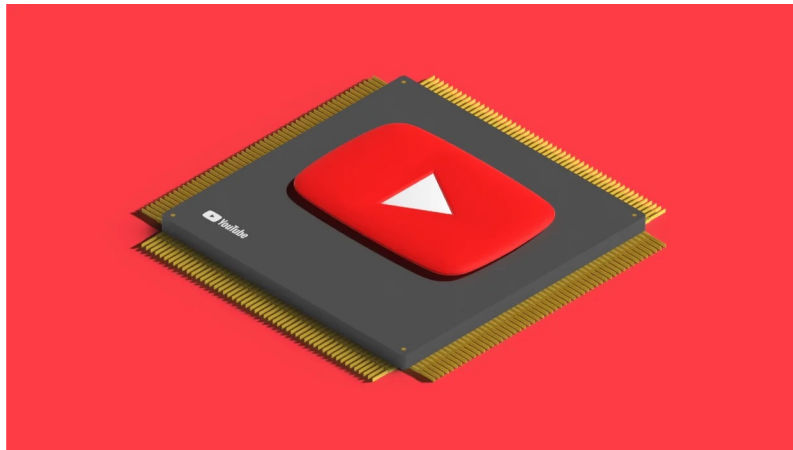
HLS-based design entry is available from all leading FPGA vendors

HLS Adoption for ASIC Design

CHIPS

Why YouTube decided to make its own video chip

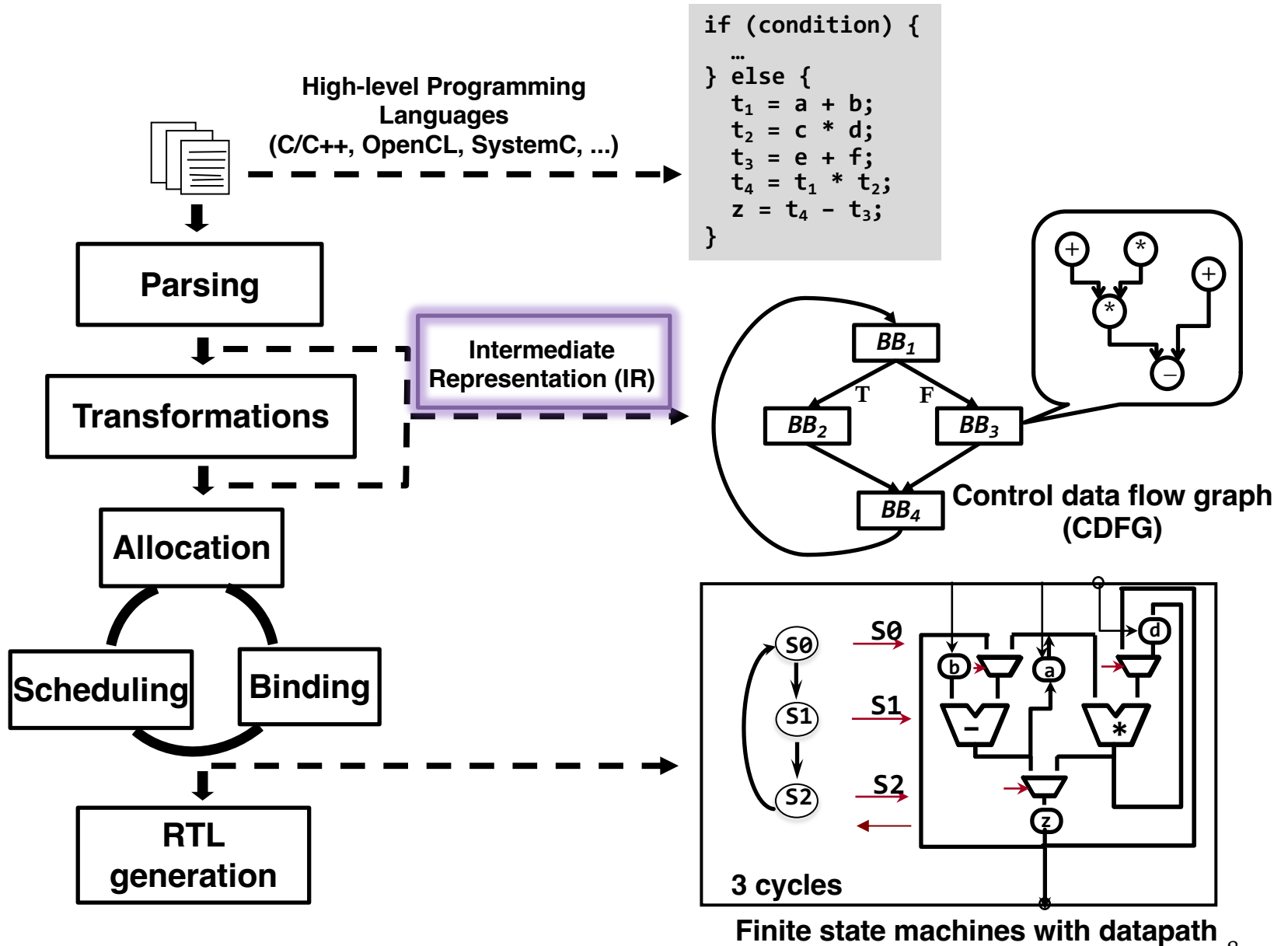
With YouTube designing its own custom chips, the company joins a growing group of big tech companies seeking to offer something unique in the data center. Operating behind the scenes, Argos made YouTube's data centers much more efficient.



YouTube Argos chip, or video-coding unit (VCU)

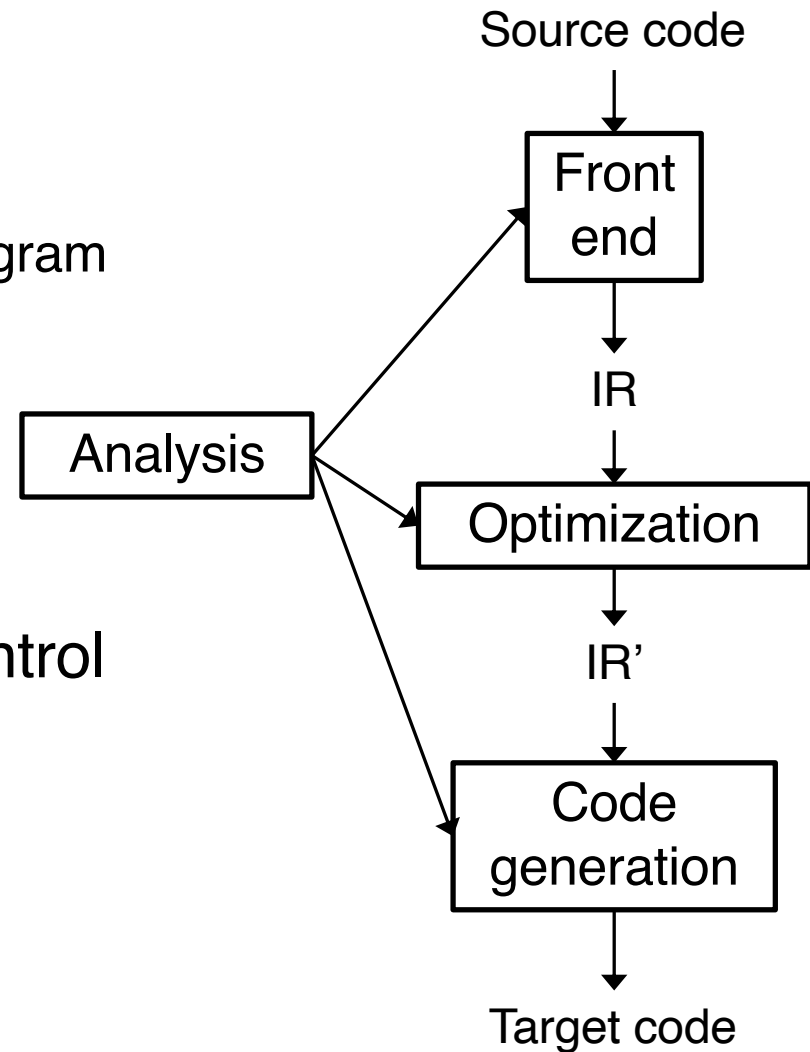
“... **Argos is a piece of hardware defined by software**, which meant that the engineers working on the chip could use what are called high-level synthesis techniques to iterate on the design much more quickly. **Google developed its own version of high-level synthesis software called Taffel that it used to help make the TPUs and the Argos processors ...**”

A Typical HLS Flow

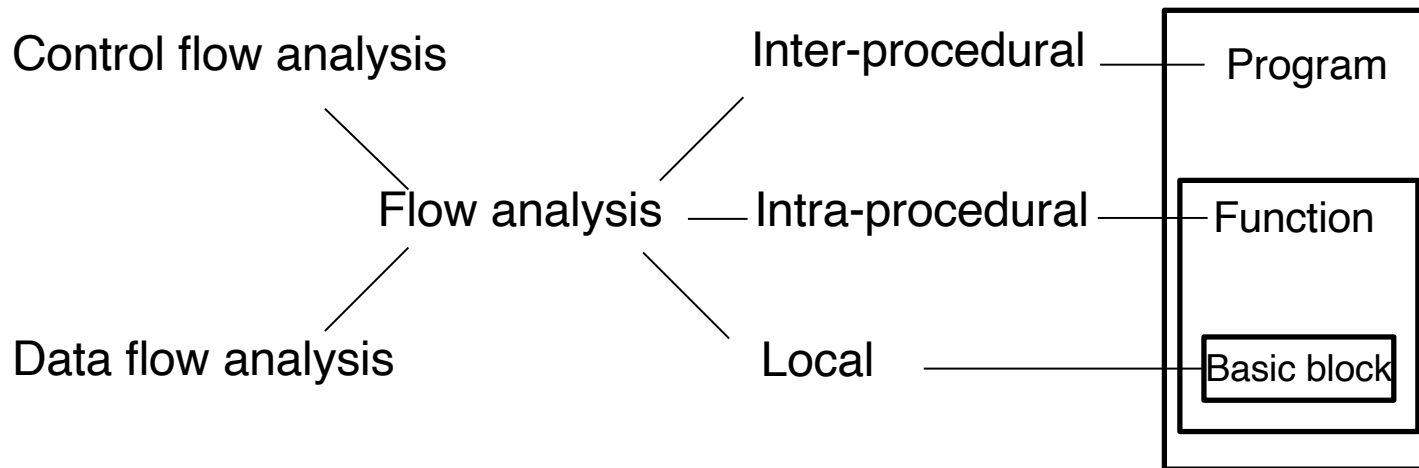


Intermediate Representation (IR)

- ▶ Purposes of creating and operating on an IR
 - Encode the behavior of the program
 - Facilitate analysis
 - Facilitate optimization
 - Facilitate retargeting
- ▶ The IR we will focus on is control data flow graph (CDFG)



Program Flow Analysis



- ▶ Control flow analysis: determine control structure of a program and build control flow graphs (**CFGs**)
- ▶ Data flow analysis: determine the flow of data values and build data flow graphs (**DFGs**)

Basic Blocks

- ▶ **Basic block:** a sequence of consecutive intermediate language statements in which flow of control can only enter at the beginning and leave at the end
 - Only the last statement of a basic block can be a branch statement and only the first statement of a basic block can be a target of a branch

Partitioning a Program into Basic Blocks

- ▶ Each basic block begins with a leader statement
- ▶ Identify leader statements (i.e., the first statements of basic blocks) by using the following rules:
 - (i) The **first statement** in the program is a leader
 - (ii) Any statement that is the **target of a branch statement** is a leader (for most intermediate languages these are statements with an associated label)
 - (iii) Any statement that **immediately follows a branch or return statement** is a leader

Example: Forming the Basic Blocks

- (1) initialize(...)
- (2) if (theta > curr) goto (7)
- (3) T = X + (Y >> i);
- (4) Y = -(X >> i) + Y;
- (5) curr = curr - ctab[i];
- (6) goto (10)
- (7) T = X - (Y >> i);
- (8) Y = (X >> i) + Y;
- (9) curr = curr + ctab[i];
- (10) X = T;
- (11) i++
- (12) if (i < STEPS) goto (2)
- (13) return X, Y

Leader statement is:

- (1) the first in the program
- (2) any that is the target of a branch
- (3) any that immediately follows a branch

Basic Blocks:

B1 (1) initialize(...)

B2 (2) if (theta > curr) goto (7)

B3 (3) T = X + (Y >> i);
(4) Y = -(X >> i) + Y;
(5) curr = curr - ctab[i];
(6) goto (10)

B4 (7) T = X - (Y >> i);
(8) Y = (X >> i) + Y;
(9) curr = curr + ctab[i];

B5 (10) X = T;
(11) i++
(12) if (i < STEPS) goto (2)

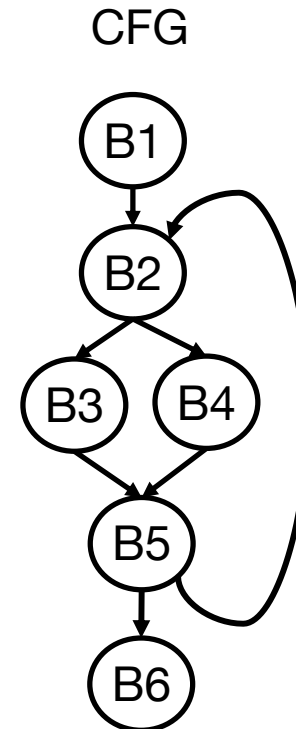
B6 (13) return X, Y

Control Flow Graph (CFG)

- ▶ A **control flow graph** (CFG), or simply a flow graph, is a directed graph in which:
 - (i) the nodes are basic blocks; and
 - (ii) the edges are induced from the possible flow of the program
- ▶ The basic block whose leader is the first intermediate language statement is called the **entry node**
- ▶ In a CFG we assume no information about data values
 - an edge in the CFG means that the program **may** take that path

Example: Control Flow Graph Formation

B1	(1) initialize(...)
B2	(2) if (theta > curr) goto (7)
B3	(3) T = X + (Y >> i); (4) Y = -(X >> i) + Y; (5) curr = curr - ctab[i]; (6) goto (10)
B4	(7) T = X - (Y >> i); (8) Y = (X >> i) + Y; (9) curr = curr + ctab[i];
B5	(10) X = T; (11) i++ (12) if (i < STEPS) goto (2)
B6	(13) return X, Y



Dominators

- ▶ A node p in a CFG **dominates** a node q if every path from the entry node to q goes through p . We say that node p is a **dominator** of node q
- ▶ The **dominator set** of node q , **DOM**(q), is formed by all nodes that dominate q
 - Each node dominates itself by definition; thus $q \in \mathbf{DOM}(q)$

Dominance Relation

- ▶ **Definition:** Let $G = (V, E, s)$ denote a CFG, where
 - V : set of nodes
 - E : set of edges
 - s : entry node andlet $p \in V, q \in V$
 - p **dominates** q , written $p \leq q$
 - also written $p \in \text{DOM}(q)$
 - p **properly (strictly) dominates** q , written $p < q$ if $p \leq q$ and $p \neq q$
 - p **immediately (or directly) dominates** q , written $p <_d q$ if $p < q$ and there is no $t \in V$ such that $p < t < q$
 - also written $p = \text{IDOM}(q)$

Example: Dominance Relation

► **Dominator sets:**

$$\text{DOM}(1) = \{1\}$$

$$\text{DOM}(2) = \{1, 2\}$$

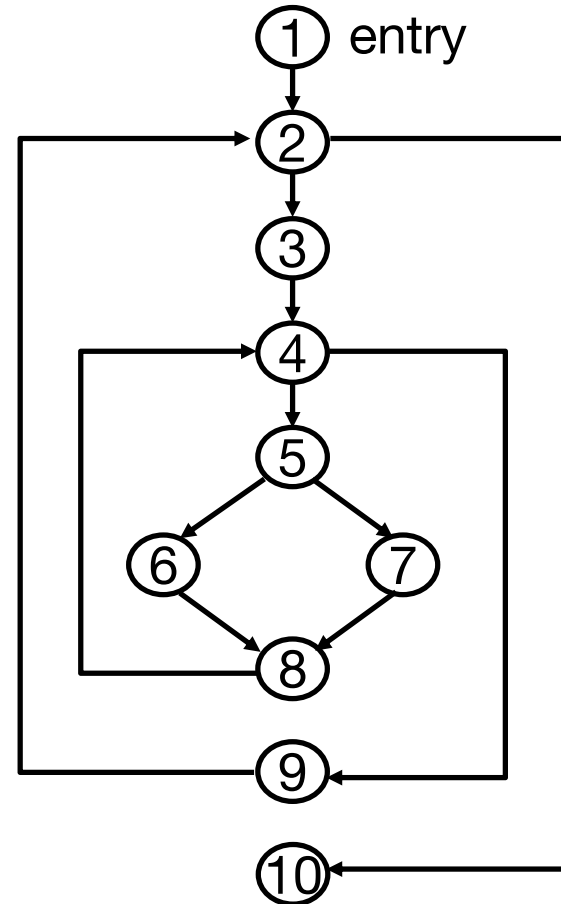
$$\text{DOM}(3) = \{1, 2, 3\}$$

$$\text{DOM}(10) = \{1, 2, 10\}$$

► **Immediate domination:**

$$1 <_d 2, 2 <_d 3, \dots$$

$$\text{IDOM}(2) = 1, \text{IDOM}(3) = 2 \dots$$



Exercise: Dominance Relationship

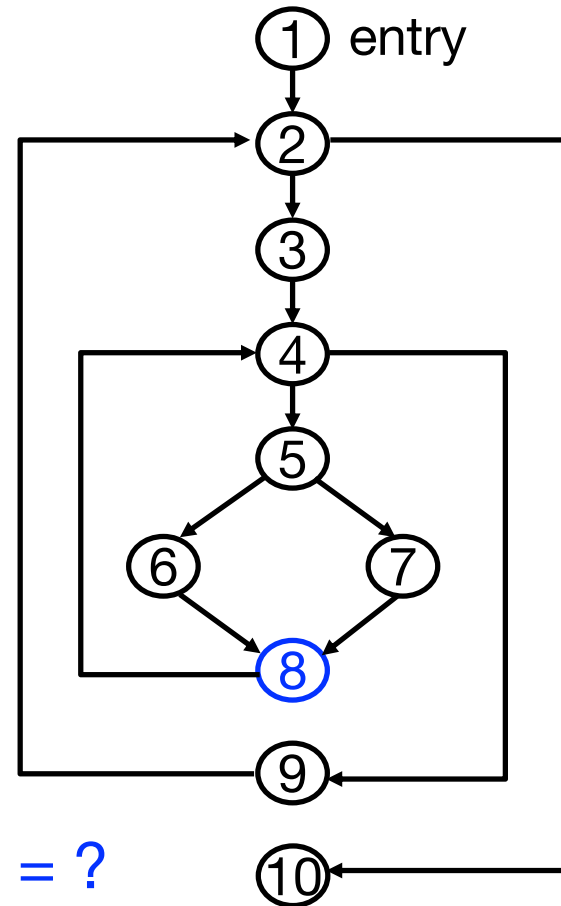
- ▶ Which of the following is TRUE
 - A CFG node strictly dominates itself
 - A CFG node always dominates its successor
 - If a node A dominates all of B's predecessors, it also dominates B

Exercise: Dominance Relationship

Assume that node P is an immediate dominator of node Q

Question: Is P necessarily a predecessor of Q in the CFG?

Answer: **NO**

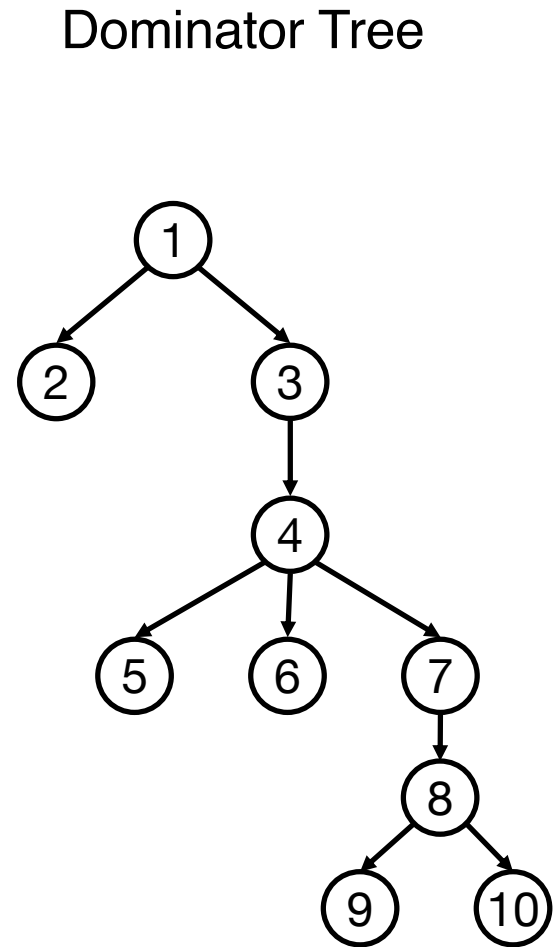
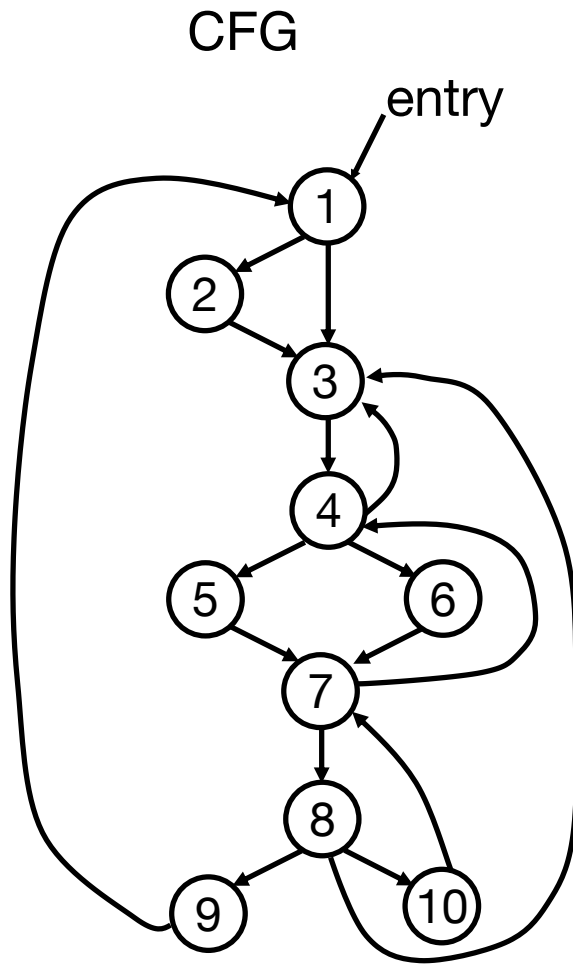


IDOM(8) = ?

Dominator Tree

- ▶ A node (basic block) N in CFG may have multiple dominators, but **only one of them will be closest to N** and be dominated by all other dominators of N
- ▶ A dominator tree is a useful way to represent the dominance relation
 - The entry node s is the root
 - **Each node in the dominator tree is the immediate dominator of its children**
 - Each node d dominates only its descendants in the tree

Example: Dominator Tree

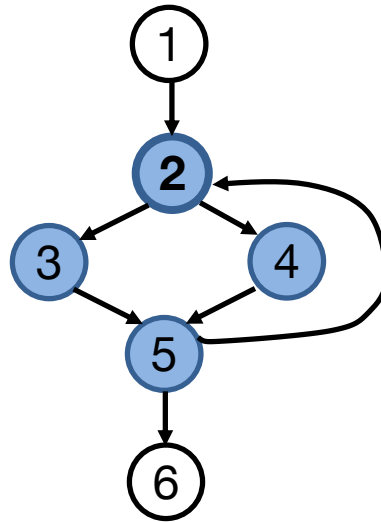


Identifying Loops

- ▶ **Motivation:** Programs spend most of the execution time in loops, therefore there is a larger payoff for optimizations that exploit loop structure
- ▶ **Goal:** Identify loops in a CFG, not sensitive to syntax of the input language
 - Create a uniform treatment for program loops written using different syntactical constructs (e.g., while, for, goto)
- ▶ **Approach:** Use a general approach based on analyzing graph-theoretical properties of the CFG

Loop Definition

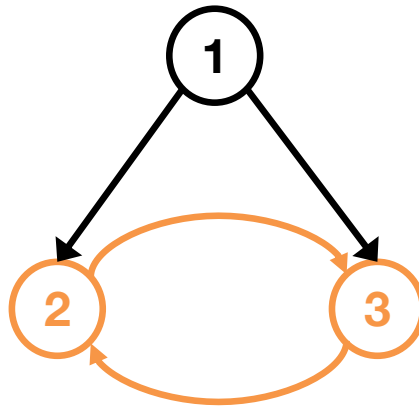
- ▶ Definition of a (natural) loop
 - A **strongly connected component** (SCC) of the CFG, with a single-entry point called the **header** which dominates all nodes in the SCC



- All nodes in blue form a loop, which is an SCC
- Node 2 is the loop **header**

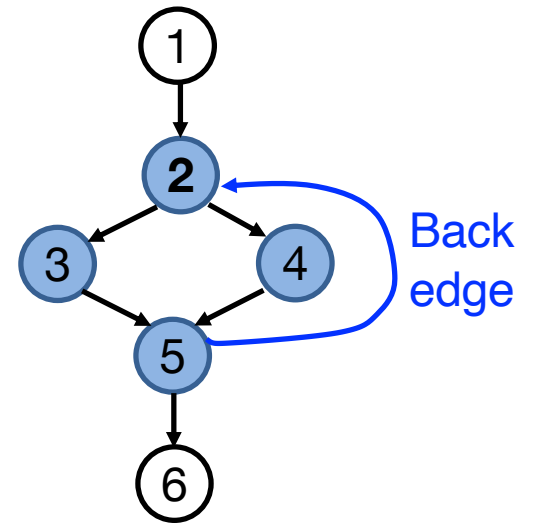
Is it a Loop?

- ▶ Question: In the CFG depicted below, do nodes 2 and 3 form an SCC, and if so, do they also form a loop?

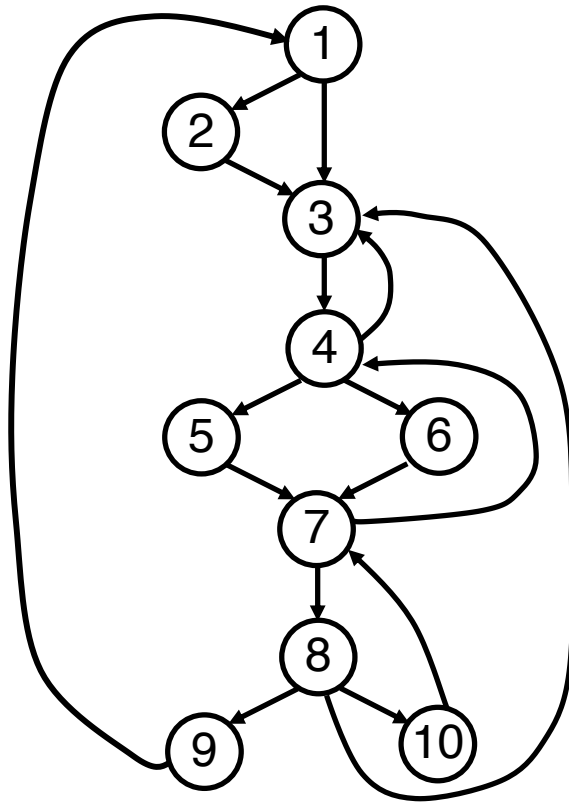


Finding Loops

- ▶ Loop identification algorithm
 1. Find an edge $B \rightarrow H$ where H dominates B
 - This edge is called a **back edge**
 - H is the header
 2. Find all nodes that (1) are dominated by H , and (2) can reach B via nodes dominated by H
 3. Add these nodes to the loop (H and B are naturally included)

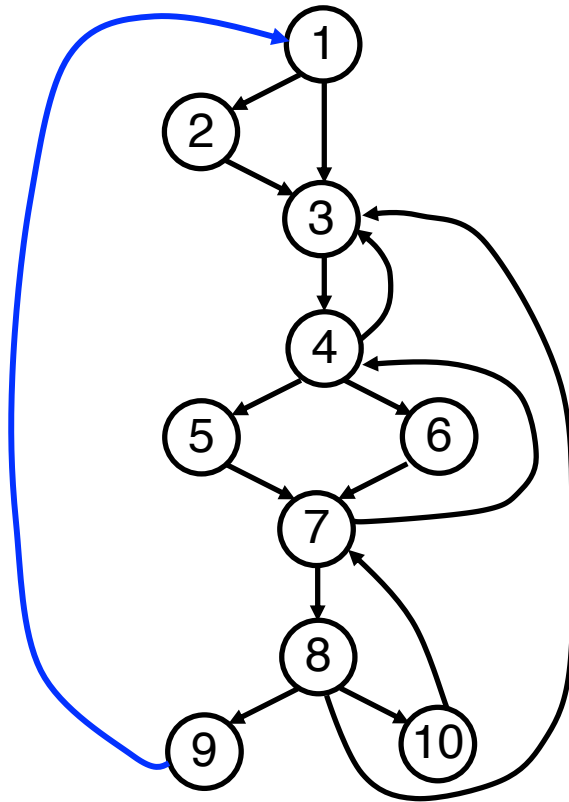


Finding Loops



Find all back edges in this graph and the natural loop associated with each back edge

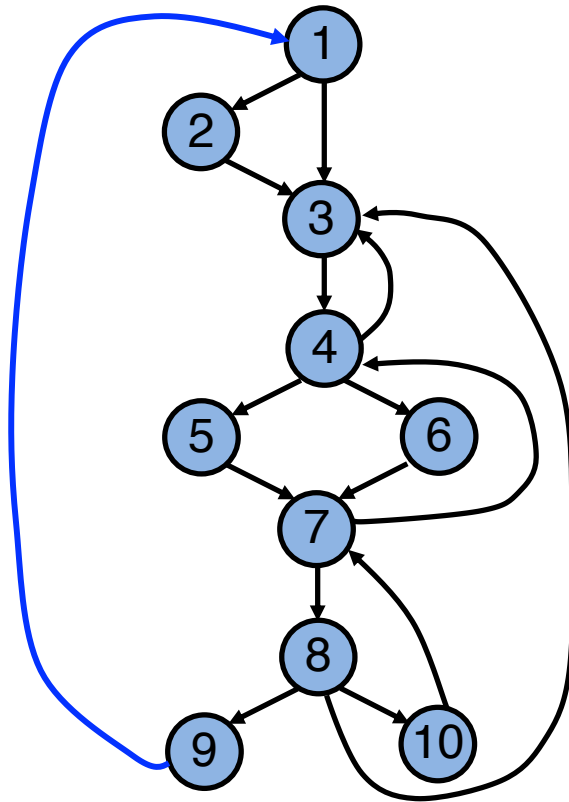
Finding Loops (1)



Find all back edges in this graph and the natural loop associated with each back edge

(9,1)

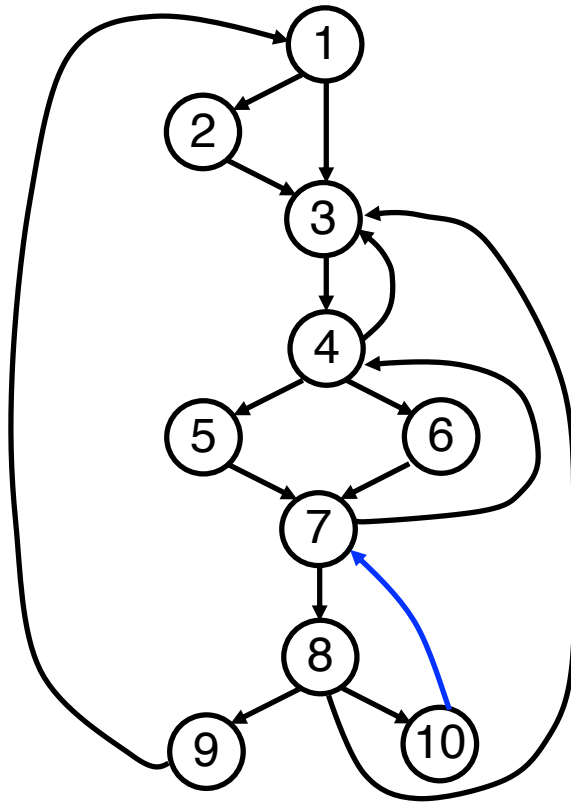
Finding Loops (1)



Find all back edges in this graph and the natural loop associated with each back edge

(9,1) Entire graph

Finding Loops (2)



Find all back edges in this graph and the natural loop associated with each back edge

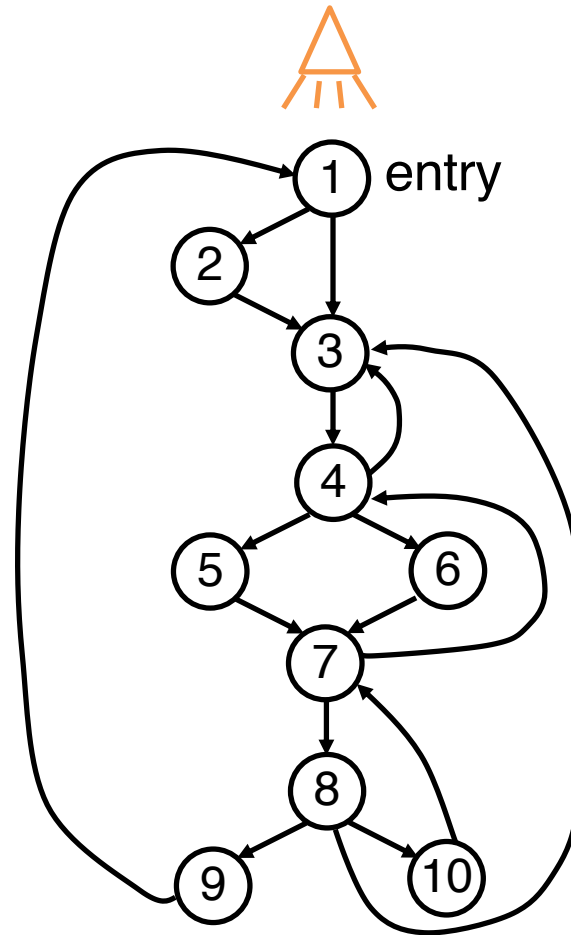
(9,1) Entire graph

(10,7)

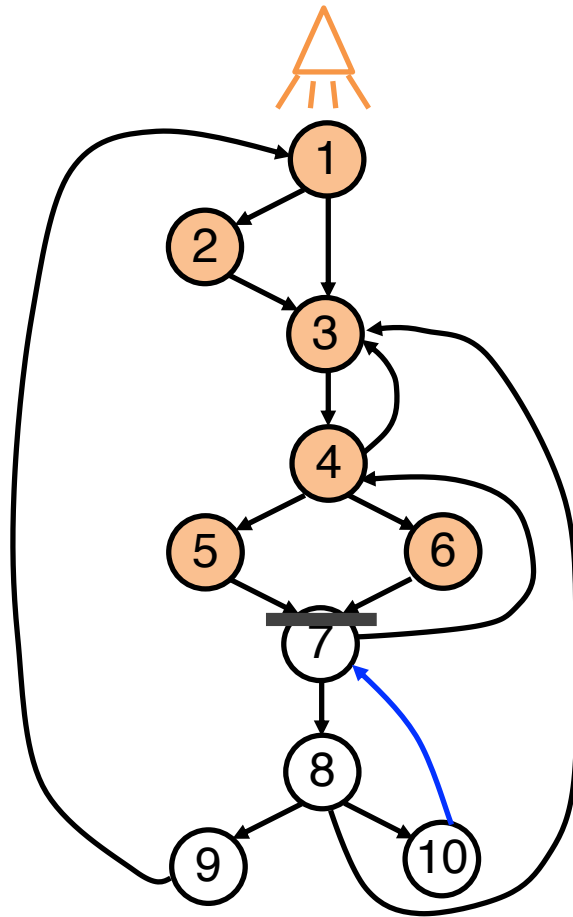
Intuition of Dominance Relation

Imagine a source of light at the entry node, and that the edges are optical fibers

To find which nodes are dominated by a given node, place an opaque barrier at that node and observe which nodes become dark



Finding Loops (2)

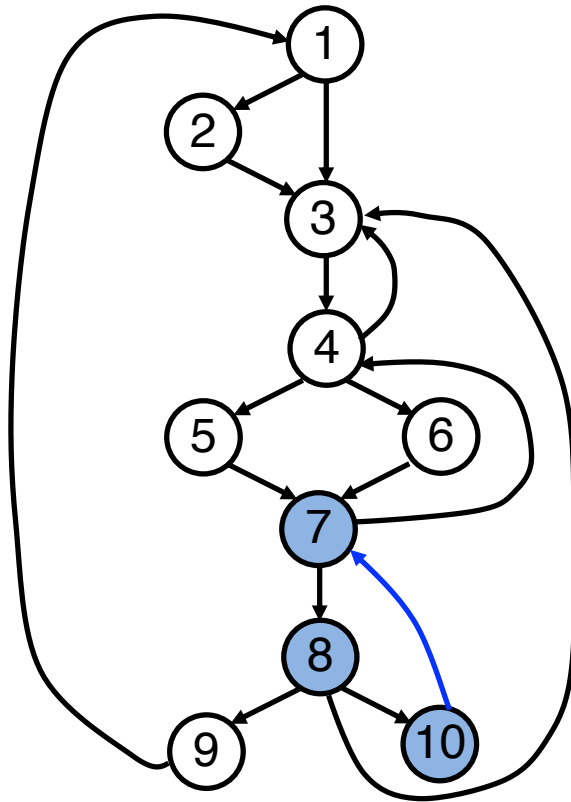


Find all back edges in this graph and the natural loop associated with each back edge

(9,1) Entire graph

(10,7)

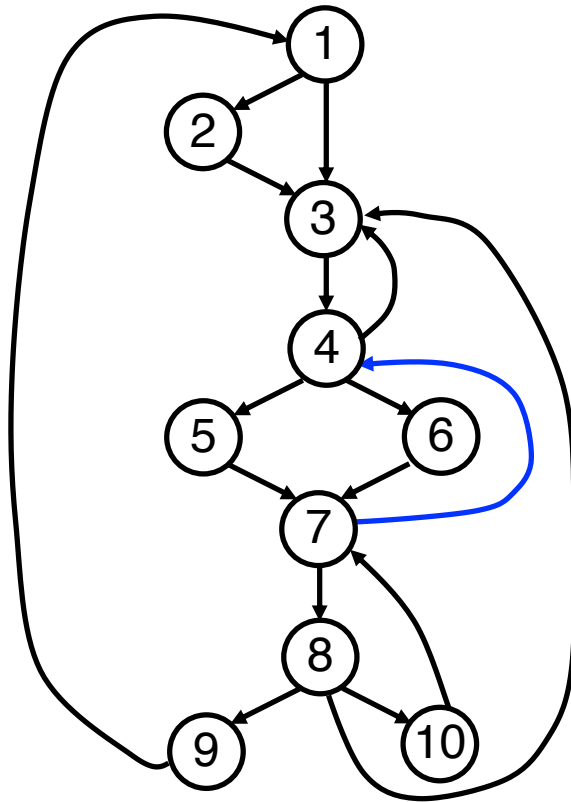
Finding Loops (2)



Find all back edges in this graph and the natural loop associated with each back edge

(9,1) Entire graph
{7,8,10}

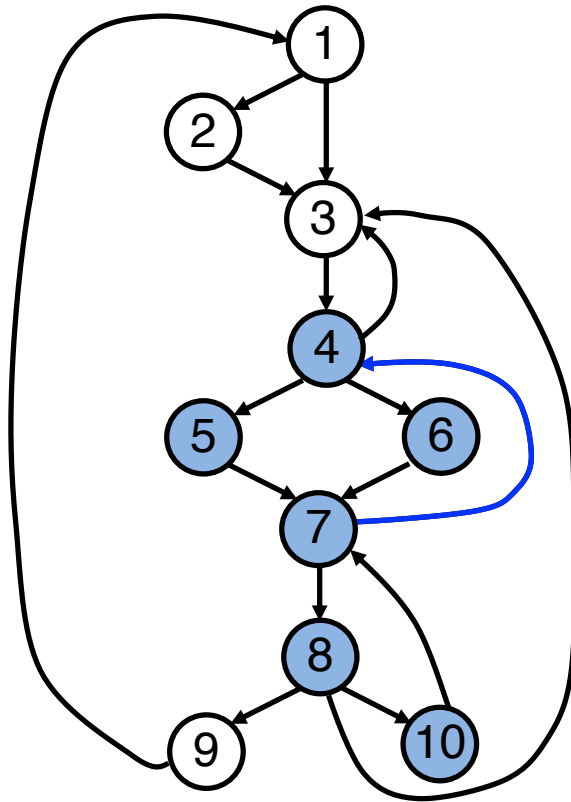
Finding Loops (3)



Find all back edges in this graph and the natural loop associated with each back edge

(9,1) Entire graph
(10,7) {7,8,10}
(7,4)

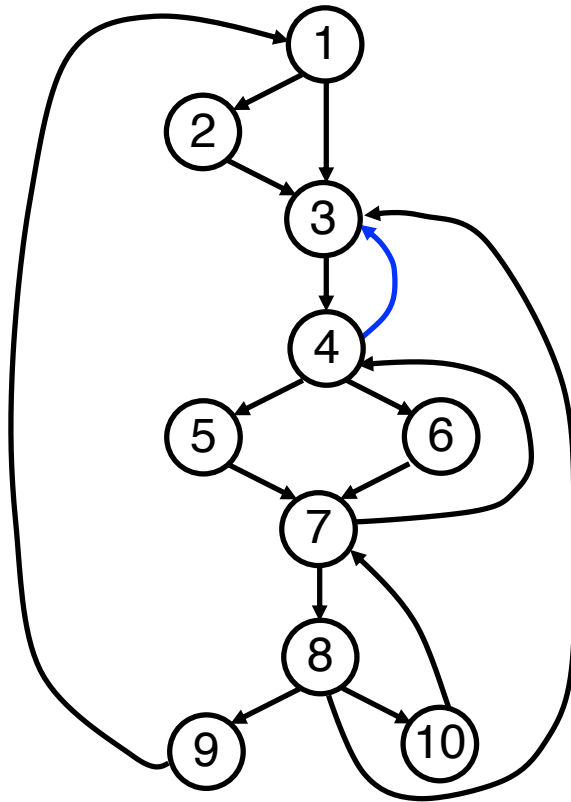
Finding Loops (3)



Find all back edges in this graph and the natural loop associated with each back edge

(9,1)	Entire graph
(10,7)	{7,8,10}
(7,4)	{4,5,6,7,8,10}

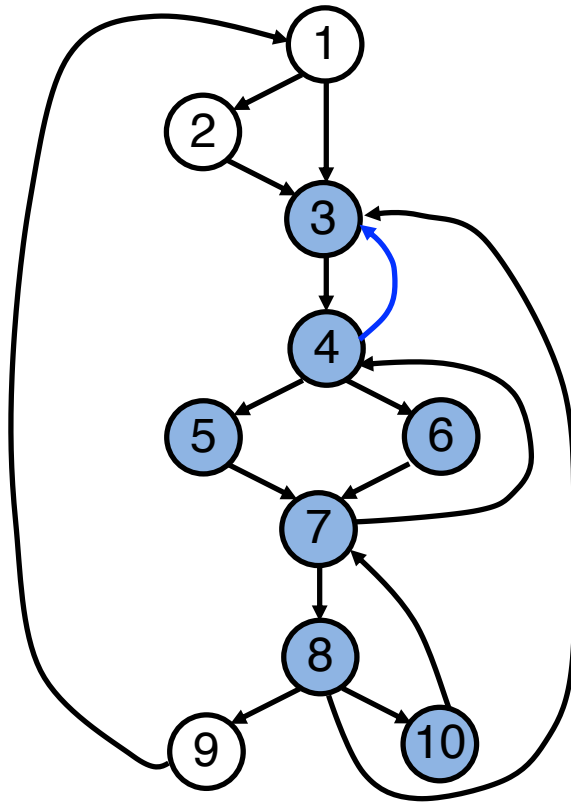
Finding Loops (4)



Find all back edges in this graph and the natural loop associated with each back edge

(9,1)	Entire graph
(10,7)	{7,8,10}
(7,4)	{4,5,6,7,8,10}
(4,3)	

Finding Loops (4)



Find all back edges in this graph and the natural loop associated with each back edge

(9,1)	Entire graph
(10,7)	{7,8,10}
(7,4)	{4,5,6,7,8,10}
(4,3)	{3,4,5,6,7,8,10}

Summary

- ▶ Basic Blocks
 - Group of statements that execute atomically
- ▶ Control Flow Graphs
 - Model the control dependences between basic blocks
- ▶ Dominance relations
 - Shows control dependences between BBs
 - Used to determine natural loops

Next Lecture

- ▶ Static single assignment (SSA)

Acknowledgements

- ▶ These slides contain/adapt materials developed by
 - Forrest Brewer (UCSB)
 - Ryan Kastner (UCSD)
 - Prof. José Amaral (Alberta)