ECE 6775
High-Level Digital Design Automation
Fall 2025

Prelim Review
HLS Design Practice

Cornell University

CSL

Announcements

- **Lab 4**: Tips on array partitioning & reshaping posted

- **Prelim** on Tuesday 10/28
  - In class, 75 mins – **arrive 5 mins early**
  - Open notes, open book, closed Internet
    - Download the latest lecture slides from the course web
  - Past quizzes posted on Ed (see pinned thread #1); click "Submit" to view the answer key)

- **Final project:** In-depth exploration of a topic related to high-level design automation
  - 3-4 students per team, 49 students overall
    - **13 teams = 10 * 4 students + 3 * 3 students**
  - Weekly meetings start in the week of 11/3
    - A teaming and meeting scheduling sheet will be posted next week

Agenda

- Prelim review
  - Overview of key lecture topics
  - Homework question discussion

- HLS design practice
  - Lab design review
  - Additional case studies

Prelim Next Tuesday (20%)

- Topics covered: lectures 01~11, 13, 14
  - Hardware specialization
  - Algorithm basics
  - FPGA
  - C-based synthesis
  - Control flow graph and SSA
  - Scheduling
  - Pipelining
  - Resource sharing

Key Topics (1)

- Algorithm basics
  - Time complexity, esp. big-O notation
  - Graphs
    - Trees, DAGs, topological sort
    - BDDs, timing analysis
- FPGAs
  - LUTs and LUT mapping
- C-based synthesis
  - Arbitrary precision and fixed-point types
  - Key HLS optimizations to improve design performance

Key Topics (2)

- Control data flow graph
  - Dominance relation
  - Loops
  - SSA

- Scheduling
  - TCS & RCS algorithms: ILP, list scheduling, SDC
    - Operation chaining, frequency/latency/resource constraints
  - Ability to devise a simple scheduling algorithm to optimize a design metric

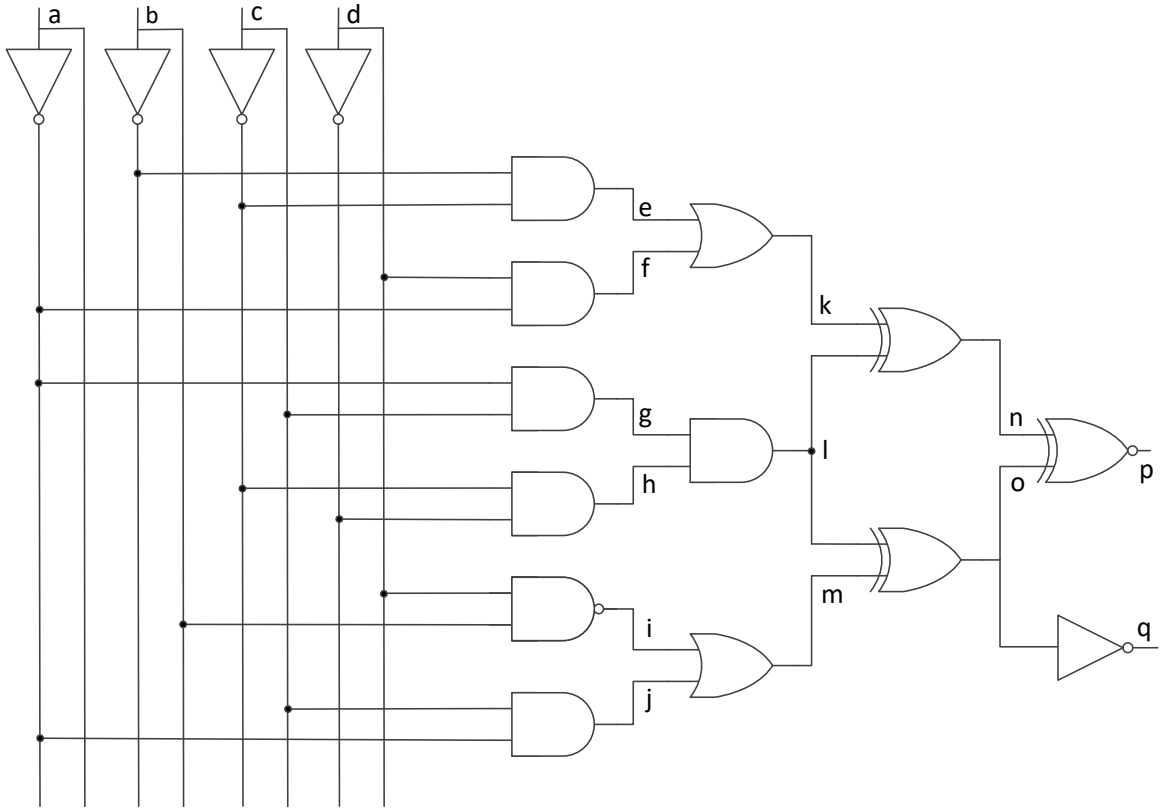
Key Topics (3)

- Pipelining
  - Dependence types
  - Ability to determine minimum II given a code snippet
    - Modulo scheduling concepts: MII, RecMII, ResMII

- Resource sharing
  - Conflict and compatibility graphs
  - Ability to determine minimum resource usage in # of functional units and/or registers, given a fixed schedule

HW1 Q5

Implement the circuit shown below using a **minimum** number of 3-input lookup tables (LUTs). Please do NOT simplify the gate-level circuit before mapping it to LUTs. Indicate your answer by creating a table similar to Table 1. Please add a new row per LUT, and use the signal names from the circuit to label the inputs (i.e., the associated cut) and output of each LUT. Mark an unused LUT input with a "–".

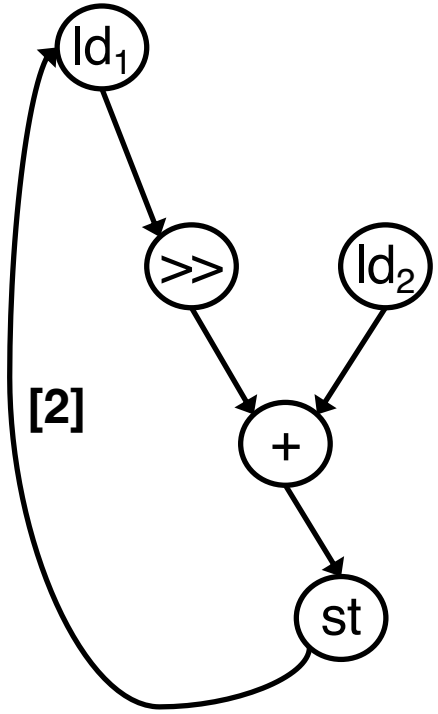


HW2 Q3

With the code snippet listed below, what is the minimum achievable II if we pipeline the loop? Please (1) draw the dependence graph for the loop, and (2) calculate both ResMII and RecMII.

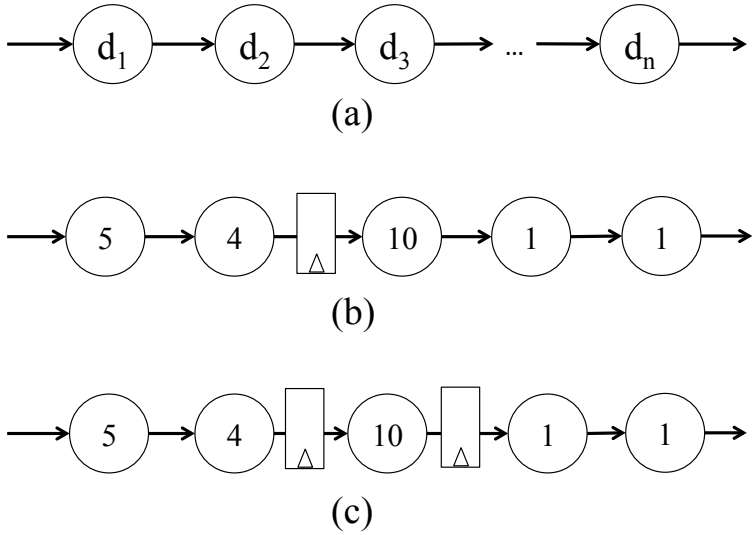
```
for (int i = 2; i < N; i++) {
  #pragma pipeline II=?
  mem[i] += (mem[i-2] >> i);
}
```

In this problem, we assume that all operations take a full cycle to execute and no combinational chaining is allowed. We also assume that the 'mem' array will be mapped to a single-ported memory block (i.e., one read/write port) without any write-through support (i.e., a memory read must happen one cycle after a write).



HW2 Q6

Given a chain of N operations $\{O_1, O_2, ..., O_N\}$ $(N > 1)$ where each operation O_i is associated with a positive integral delay value of d_i $(d_i \leq 10ns)$, our goal is to automatically place (or insert) one or multiple registers into this chain to minimize the clock period of the circuit (i.e., maximize the operating frequency).



HW2 Practice

To linearize **R = Y AND Z**

R >= 0
R >= Y + Z – 1
R <= Y
R <= Z

To form an integer linear programming (ILP) model for the constrained scheduling problem, we have learned to use a binary decision variable $X(i,k)$ to indicate if operation i starts at cycle k, where $1 \leq k \leq L$ with L being the maximum schedule latency.

In this problem, we will make use of these schedule variables to test if the output value of a given operation v is live at a specific cycle s ($1 \leq s \leq L$). Here we make the following assumptions:

- The output value of any operation v (other than the primary outputs that do not have successors) is used by exactly ONE other operation, denoted as $USE[v]$.
- We also assume that each operation takes exactly one single full cycle and no chaining is allowed.

The definition of the liveness of a value is illustrated in the Figure 3.

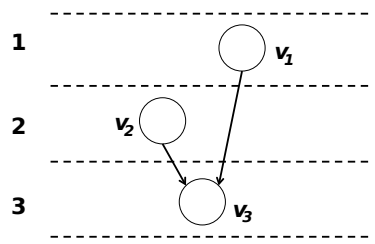


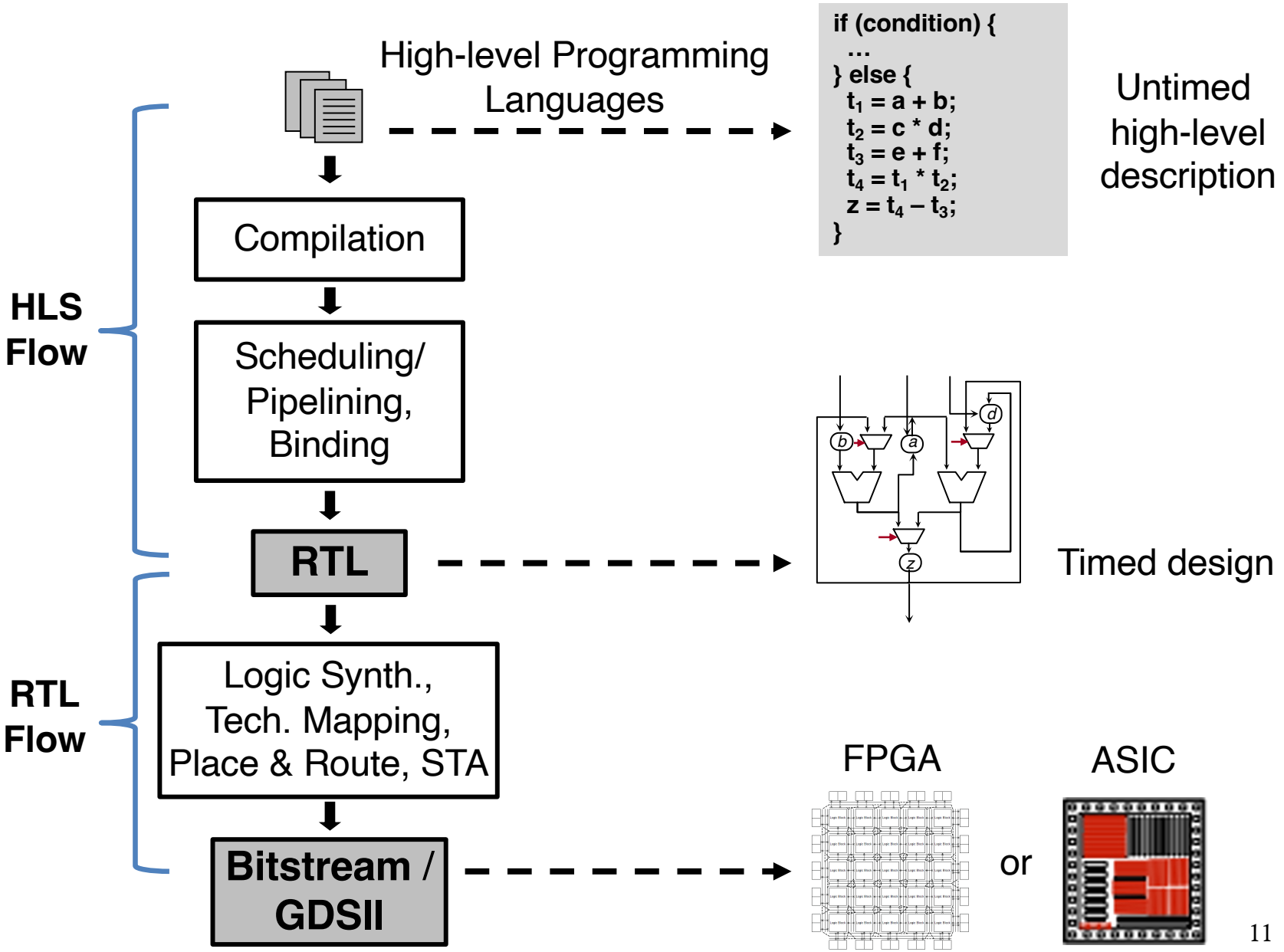
Figure 3: A scheduled graph with three operations.

(**a**) Can you introduce a **derived** binary variable $Y(v,s)$ to indicate if an operation v is scheduled before cycle s (excluding s)? Hint: Make use of the relevant X's.

(**b**) Can you introduce another derived binary variable $Z(v,s)$ to indicate if an operation v is scheduled at or after cycle s? Hint: Make use of Y.

(**c**) Finally, can you make use of the variables introduced from (a) and (b) to derive binary variable $R(v,s)$, which indicates if the output value of v is live at cycle s. Hint: You may need to introduce additional linear constraint(s) between Y, Z, and/or R. You may also make use of $USE[v]$ based on the assumptions stated.

Recap: A Typical HLS Design Flow



Essence of Accelerator Design with HLS

- To achieve high performance:

  Run as many processing elements (PEs) in parallel as possible

  → *unrolling* (more PEs), *pipelining* (higher PE utilization)

  Challenge: Keep the PEs continuously fed with data

- Common bottlenecks:

  Limited memory bandwidth, on-chip or off-chip, e.g., each BRAM has only two ports

  → *array partitioning* (more ports), *array reshaping* (wider access per port), *data reuse* (fewer off-chip accesses)

  Limited on-chip capacity (memory or compute):

  → *tiling*, applies to loop and data

  Carried dependence, i.e., recurrence:

  → *reordering* of compute (e.g., loop reorder) can help in some cases

Case Study: CORDIC

```
void cordic(theta_type theta, cos_sin_type &s, cos_sin_type &c) {
  double K_const = 0.607252935008881256169 4;
  theta_type current = 0;
  cos_sin_type X = K_const; Y = 0, T;
  for (int step = 0; step < 20; step++) {
    if (theta > current) {
      T = X - (Y >> step);
      Y = (X >> step) + Y;
      X = T;
      current = current + cordic_ctab[step];
    } else {
      T = X + (Y >> step);
      Y = -(X >> step) + Y;
      X = T;
      current = current - cordic_ctab[step];
    }
  }
  s = Y;
  c = X;
}
```

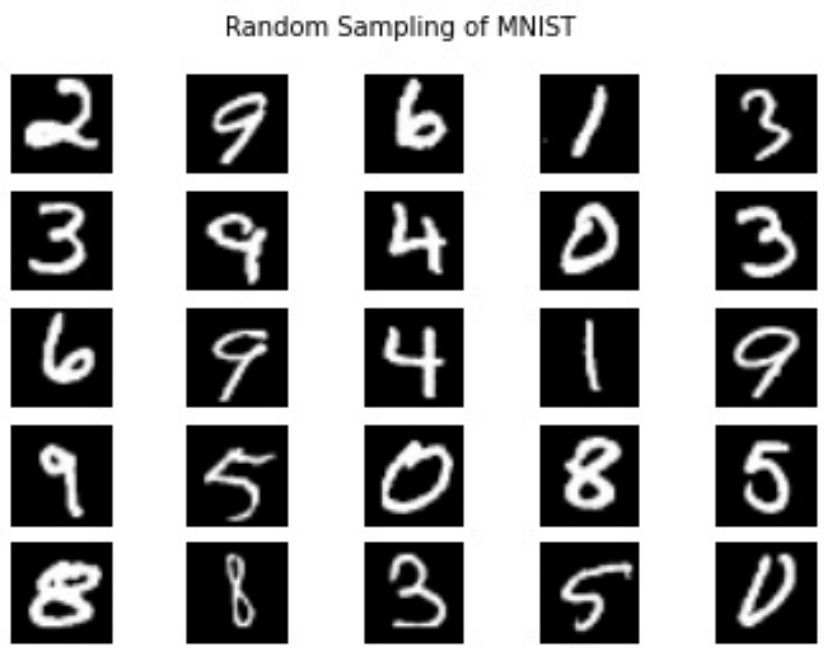
Pipeline the whole function (loop inside automatically unrolled)

II=1 means one CORDIC per cycle

Unroll or pipeline this loop?

Case Study: Digit Recognition

- Use a simple machine learning algorithm to recognize handwritten digits
  - 2000 training instances per digit
  - Each training/test instance is a 7x7 bitmap after downsampling



MNIST dataset: http://yann.lecun.com/exdb/mnist/

K-Nearest-Neighbor (KNN) Implementation

```
bit4 digitrec( digit input )
{
  #include "training_data.h"
  // This array stores K minimum distances per training set
  bit6 knn_set[10][K_CONST];
  // Initialize the knn set
  for ( int i = 0; i < 10; ++i )
    for ( int k = 0; k < K_CONST; ++k )
      // Note that the max distance is 49
      knn_set[i][k] = 50;
```

Main compute loop

```
L2000: for ( int i = 0; i < TRAINING_SIZE; ++i ) {
  L10:   for ( int j = 0; j < 10; j++ ) {

    // Read a new instance from the training set
    digit training_instance = training_data[j * TRAINING_SIZE + i];
    // Update the KNN set
    update_knn( input, training_instance, knn_set[j] );
  }
}
```

Assuming 10 cycles per innermost loop (L10)

~200K cycles by default without optimizations

10x Speedup through Parallel Processing

```
bit4 digitrec( digit input )
{
  #include "training_data.h"
  // This array stores K minimum distances per training set
  bit6 knn_set[10][K_CONST];
  // Initialize the knn set
  for ( int i = 0; i < 10; ++i )
    for ( int k = 0; k < K_CONST; ++k )
      // Note that the max distance is 49
      knn_set[i][k] = 50;
```

Unroll inner loop completely

```
  L2000: for ( int i = 0; i < TRAINING_SIZE; ++i ) {
   L10:    for ( int j = 0; j < 10; j++ ) {

      // Read a new instance from the training set
      digit training_instance = training_data[j * TRAINING_SIZE + i];
      // Update the KNN set
      update_knn( input, training_instance, knn_set[j] );
    }
  }
```

Partition training set into 10 banks

10 instances of "update_knn" running in parallel

~20K cycles after parallelization

Further Speedup through Pipelining

```
bit4 digitrec( digit input )
{
  #include "training_data.h"
  // This array stores K minimum distances per training set
  bit6 knn_set[10][K_CONST];
  // Initialize the knn set
  for ( int i = 0; i < 10; ++i )
    for ( int k = 0; k < K_CONST; ++k )
      // Note that the max distance is 49
      knn_set[i][k] = 50;

  L2000: for ( int i = 0; i < TRAINING_SIZE; ++i ) {
    L10:   for ( int j = 0; j < 10; j++ ) {

      // Read a new instance from the training set
      digit training_instance = training_data[j * TRAINING_SIZE + i];
      // Update the KNN set
      update_knn( input, training_instance, knn_set[j] );
    }
  }
```

Pipeline outer loop

Outer loop (L2000) pipelined to II=1

~2K cycles after pipelining

Array Partitioning Caveats

Example: Array partitioning through *constant indices* (after unrolling)

Original code

```
for (int i = 0; i < N; ++i)
  for (int k = 0; k < 8; ++k)
    sum += A[k][i];
```

partition "A" to 8 sub-arrays on dimension "k"
without unrolling the inner loop

Transformed loop body

```
switch (k) {
  case 0: sum += A_0[i]
  case 1: sum += A_1[i]
  ...
  case 7: sum += A_7[i]
}
```

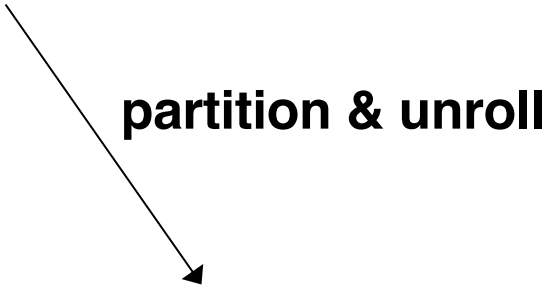
Inefficient design
The switch-case statement will be synthesized into (large) multiplexers in RTL

Array Partitioning Caveats

Example: Array partitioning through *constant indices* (after unrolling)

Original code

```
for (int i = 0; i < N; ++i)
  for (int k = 0; k < 8; ++k)
    sum += A[k][i];
```



Transformed code

```
for (int i = 0; i < N; ++i) {
  sum += A_0[i];
  sum += A_1[i];
  ...
  sum += A_7[i];
}
```

Efficient design

After unrolling inner loop, the resulting indices on the "k" dimension become constant, which simplify the logic after array partitioning

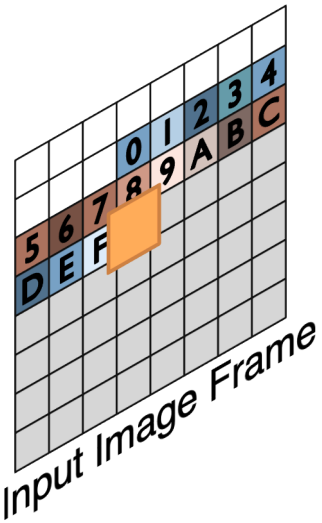
Case Study: Revisiting 3x3 Convolution

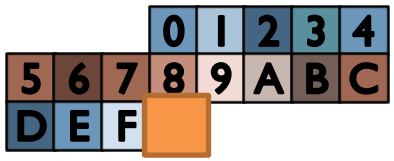
```
for (r = 1; r < H; r++)
   for (c = 1; c < W; c++) {
     #pragma HLS pipeline II=?
     for (i = 0; i  < 3; i++)
       for (j = 0; j < 3; j++)
         out += img[r+i-1][c+j-1] * f[i][j];
     out[r][c] = out;
    }
```

- Inner loops “i" & j are automatically unrolled
- The 3x3 filter array “f” is automatically partitioned into 9 registers after unrolling
- The entire input image “img” is stored in an on-chip buffer with **two read ports**

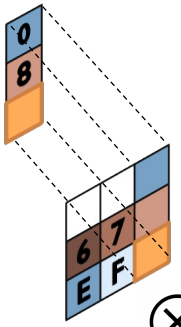
ResMII = 5 RecMII = 1

Achieving II=1 for 3x3 Convolution using a Line Buffer and Shift Registers





Pixels in **line buffer**
(stores 2 lines using on-chip SRAMs)



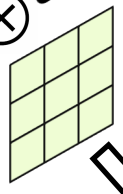
1. **Push 3 pixels into shift registers** –
   1 new pixel +
   2 from line buffer

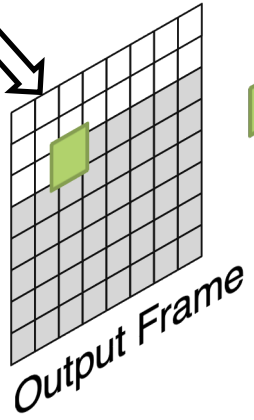
2. **Update line buffer**
   by removing the oldest pixel and shifting in the new one

Shift registers

⊗ *Convolution (conv)*

Filter weights





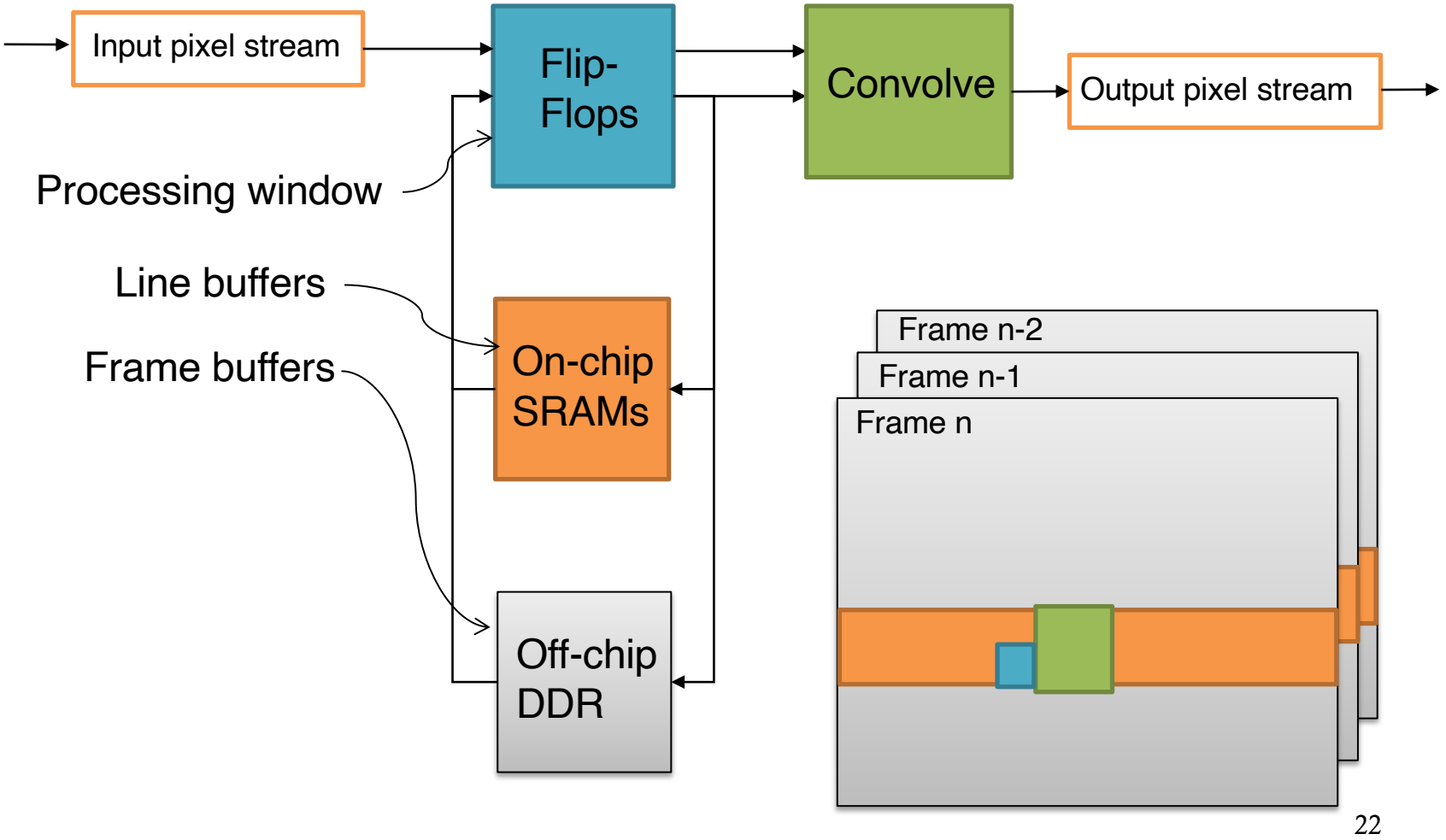
Output pixel produced by conv

New pixel read from frame buffer in main memory (DRAM)

Line Buffer + Shift Registers:
a custom “cache” + a custom “register file”

Resulting Specialized Memory Hierarchy

- Memory architecture customized for convolution



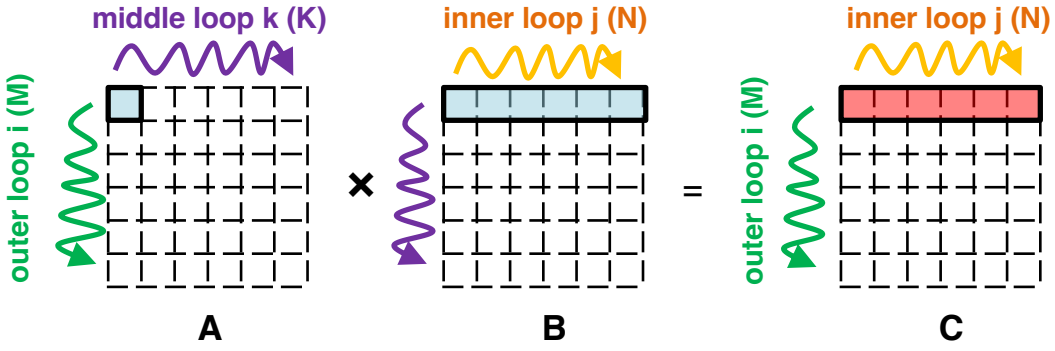
HLS Code Snippet

```
1   LineBuffer<2,C,pixel_t> linebuf;
2   Window<3,3,pixel_t> window;
3   for (int r = 1; r < R+1; r++) {
4     for (int c = 1; c < C+1; c++) {
5       #pragma HLS pipeline II=1
6       pixel_t new_pixel = img[r][c];
7       // Update shift window
8       window.shift_left();
9       if (r < R && c < C) {
10        for (int i = 0; i < 2; i++ )
11          window.insert(buf[i][c]);
12      }
13      else { // zero padding
14        for (int i = 0; i < 2; i++)
15          window.insert(0);
16      }
17      window.insert(new_pixel);
18      // Update line buffer
19      linebuf.shift_up(c);
20      if (r < R && c < C)
21        linebuf[1].insert(c, new_pixel);
22      else // Zero padding
23        linebuf[1].insert(c, 0);
24      // Perform 3x3 convolution
25      out[r-1][c-1] = convolve(window, weights);
26    }
27  }
```

Case Study: Revisiting MatMul Pipeline

- The row-wise product approach performs a sequence of scalar-vector products to produce *one output row*
  - An additional buffer is added to store the intermediate results (i.e., c_vec)

MatMul via row-wise product



$$C[i,:] = \sum_k A[i,k] \cdot B[k,:]$$

```
for (int i = 0; i < M; i++) {
  float C_vec[N];
  for (int j = 0; j < N; j++)
    C_vec[j] = 0.0;

  for (int k = 0; k < K; k++) {
    for (int j = 0; j < N; j++) {
      #pragma pipeline II=??
      C_vec[j] += A[i, k] * B[k, j];
    }
  }
  for (int j = 0; j < N; j++)
    C[i, j] = C_vec[j];
}
```

Case Study: Revisiting Conv Layer Pipeline

```
9           for (ki=0; ki<K; ki++) {
10            for (kj=0; kj<K; kj++) {
5               for (trr=row; trr<min(row+Tr, R); trr++) {
6                 for (tcc=col; tcc<min(col+Tc, C); tcc++) {
                    #pragma HLS pipeline
7                   for (too=to; too<min(to+To, O); too++) {
                      #pragma HLS unroll
8                     for (tii=ti; tii<min(ti+Ti, I); tii++) {
                        #pragma HLS unroll
                        output_fm[too][trr][tcc] +=
                        weights[too][tii][ki][kj]*input_fm[tii][S*trr+ki][S*tcc+kj];
            }}}}}}
```

Parallelize across input (Ti) and output (To) channels

Number of cycles to execute the above loop nest

$$\approx K \times K \times Tr \times Tc + L \approx Tr \times Tc \times K^2$$

L is the pipeline depth (i.e., # of pipeline stages), assume II=1

A (Simplified) Loopy Form of Processor Pipeline

```
for ( pc++ ) {
  inst = fetch( pc )
  { rs, rt, rd } = decode( inst )
  result = execute( reg[rs], reg[rt] )
  mem( result, reg[rt] )
  reg[rd] = writeback( result )
}
```

Next Tuesday

- In-Class Prelim